**US Army Corps
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User's Guide for the Sigma Stretched Version of CH3D-WES

A Three-Dimensional Numerical Hydrodynamic, Salinity, and Temperature Model

by *Raymond S. Chapman, Ray Chapman & Associates*
Billy H. Johnson, S. Rao Vemulakonda, WES

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User's Guide for the Sigma Stretched Version of CH3D-WES

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by Raymond S. Chapman

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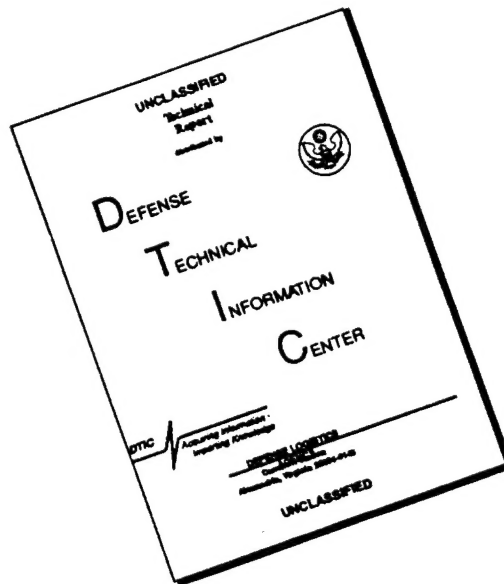Vicksburg, MS   39180

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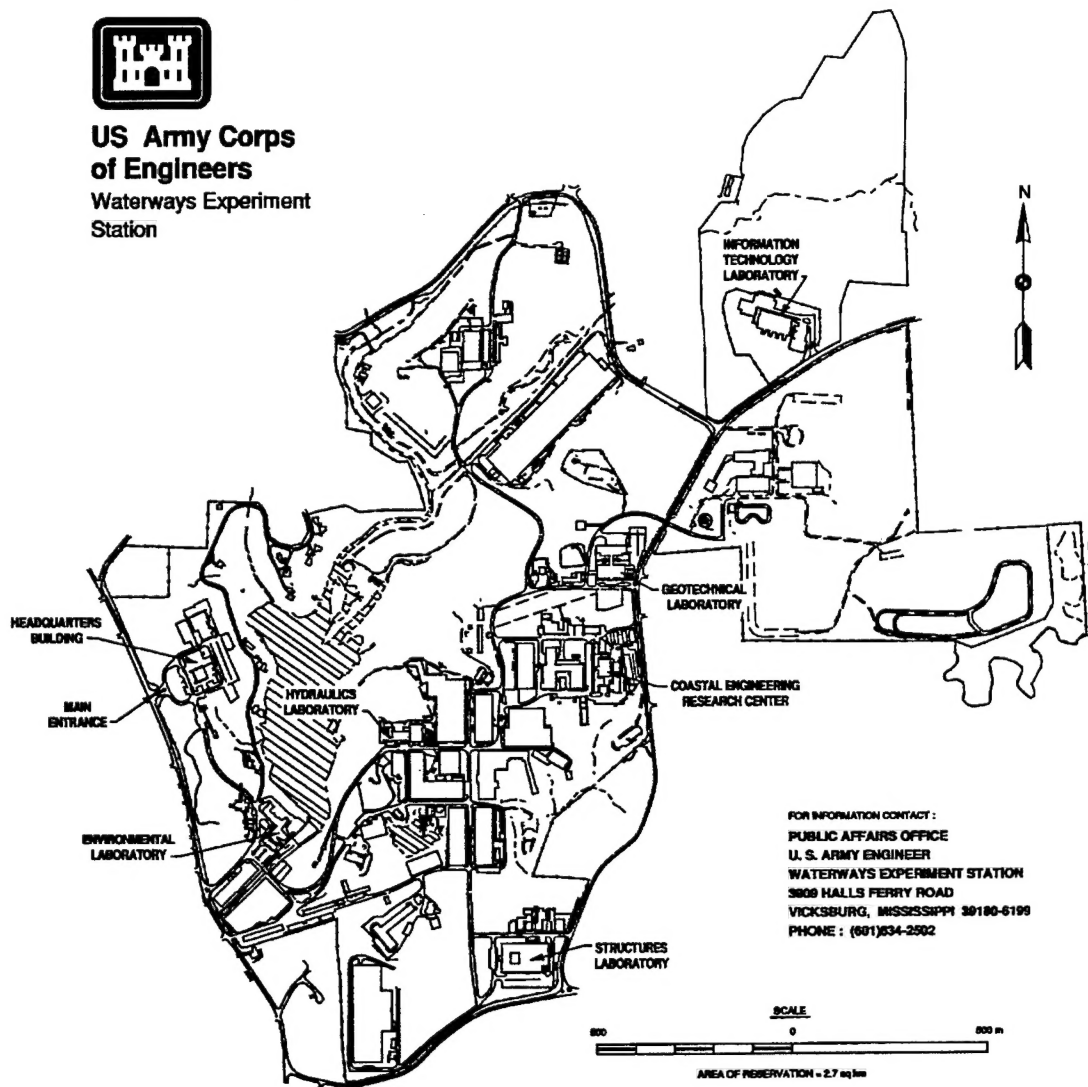
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STRUCTURES
LABORATORY

SCALE

800   0   800 m

AREA OF RESERVATION = 2.7 sq km

Contents

Preface

This study was conducted during 1995-1996 by Drs. Ray Chapman of Ray Chapman & Associates, Vicksburg, MS, Billy H. Johnson, Hydraulics Laboratory (HL), U.S. Army Engineer Waterways Experiment Station (WES); and S. Rao Vemulakonda, Coastal Oceanography Branch, Research Division, Coastal Engineering Research Center (CERC), WES, under the Numerical Model Maintenance program of the Headquarters, U.S. Army Corps of Engineers, under the general supervision of Mr. Richard A. Sager, Acting Director, HL; Mr. Robert F. Athow, Acting Assistant Director, HL; Dr. Martin C. Miller, Chief, Coastal Oceanography Branch, CERC; Mr. H. Lee Butler, Chief, Research Division; CERC; Mr. Charles C. Calhoun, Jr., Assistant Director, CERC; and Dr. James R. Houston, Director, CERC.

During the preparation and publication of this report, Director of WES was Dr. Robert W. Whalin. Commander was COL Bruce K. Howard, EN.

The report should be cited as follows:

1 Introduction

This user's guide presents detailed discussions of the theoretical aspects of the sigma version of CH3D-WES (Curvilinear Hydrodynamics in 3-Dimensions - Waterways Experiment Station). The governing equations are presented in both Cartesian and boundary-fitted form along with a discussion of boundary conditions and solution techniques. Particular attention is paid to addressing recent improvements to the representation of the horizontal momentum diffusion terms and to the prediction of vertical turbulent transport. In addition, the structure of the computer code is discussed via a description of the order and function of each subroutine. Finally, the required input to operate the model and output files generated by the model are discussed.

2 Sigma Stretched CH3D-WES Hydrodynamic Model

The numerical hydrodynamic model CH3D-WES exists in both a Z-grid and a sigma stretched version for representation of the vertical dimension. The Z-grid version was developed during a study on Chesapeake Bay and is documented in Johnson, et al. (1991b). The basic sigma stretched model was developed by Sheng (1986) for WES but has been extensively modified. These modifications have consisted of implementing different basic numerical formulations of the governing equations as well as substantial recoding of the model to provide more efficient computing. In particular, two recent modifications presented in this report include the incorporation of a compact form of the horizontal momentum diffusion terms, and a two-equation vertical (k-ϵ) turbulence model. As its name implies, CH3D-WES makes hydrodynamic computations on a curvilinear or boundary-fitted planform grid. Physical processes impacting circulation and vertical mixing that are modeled include tides, wind, density affects (salinity and temperature), freshwater inflows, turbulence, and the effect of the earth's rotation.

The boundary-fitted or curvilinear coordinate feature of the model in the horizontal dimensions provides the grid resolution enhancement necessary to adequately represent deep navigation channels and irregular shoreline configurations of the flow system. The curvilinear grid also permits adoption of accurate and economical grid schematization software. The solution algorithm employs both an external mode, consisting of vertically averaged equations which provide a solution for the free surface displacement and vertically averaged velocities, and an internal mode. The deviation of the horizontal components of the full 3D velocity from the vertically-averaged velocity components are computed in the internal mode and then added to the vertically averaged components to yield the full 3D horizontal components. In addition, the vertical component of the 3D velocity field and the 3D salinity and temperature fields are computed in the internal mode.

Governing Equations

The governing partial differential equations are based on the following

assumptions: a) the hydrostatic pressure distribution adequately describes the vertical distribution of fluid pressure. b) the Boussinesq approximation is appropriate. c) the eddy viscosity approach adequately describes turbulent mixing in the flow.

The basic equations for an incompressible fluid in a right-handed Cartesian coordinate system (x,y,z) are (Johnson et al., 1991b):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = fv - \frac{1}{\rho_o}\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[A_h\frac{\partial u}{\partial x}\right]$$

$$+ \frac{\partial}{\partial y}\left[A_h\frac{\partial u}{\partial y}\right] + \frac{\partial}{\partial z}\left[A_v\frac{\partial u}{\partial z}\right] \tag{2}$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -fu - \frac{1}{\rho_o}\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[A_h\frac{\partial v}{\partial x}\right]$$

$$+ \frac{\partial}{\partial y}\left[A_h\frac{\partial v}{\partial y}\right] + \frac{\partial}{\partial z}\left[A_v\frac{\partial v}{\partial z}\right] \tag{3}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{4}$$

$$\frac{\partial T}{\partial t} + \frac{\partial uT}{\partial x} + \frac{\partial vT}{\partial y} + \frac{\partial wT}{\partial z} = \frac{\partial}{\partial x}\left[K_h\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_h\frac{\partial T}{\partial y}\right]$$

$$+ \frac{\partial}{\partial z}\left[K_v\frac{\partial T}{\partial z}\right] \tag{5}$$

$$\frac{\partial S}{\partial t} + \frac{\partial uS}{\partial x} + \frac{\partial vS}{\partial y} + \frac{\partial wS}{\partial z} = \frac{\partial}{\partial x}\left[K_h\frac{\partial S}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_h\frac{\partial S}{\partial y}\right]$$

$$+ \frac{\partial}{\partial z}\left[K_v\frac{\partial S}{\partial z}\right] \tag{6}$$

$$\rho = \rho \ (T,S) \tag{7}$$

where

(u,v,w) = velocities in (x,y,z) directions

t = time

f = Coriolis parameter defined as $2\Omega \sin \phi$

where

Ω = rotational speed of the earth

ϕ = latitude

ρ = density

p = pressure

A_h, K_h = horizontal turbulent eddy viscosity/diffusivity coefficients

A_v, K_v = vertical turbulent eddy viscosity/diffusivity coefficients

g = gravitational acceleration

T = temperature

S = salinity.

Equation 4 implies that vertical accelerations are negligible and thus the pressure is hydrostatic. Various forms of the equation of state can be specified for Equation 7. In the present model, the formulation given below is used:

$$\rho = P/(\alpha + 0.698P) \tag{8}$$

where

ρ = density in grams per cubic centimeter

$P = 5890 + 38T - 0.375T^2 + 3S$

$$\alpha = 1779.5 + 11.25T - 0.0745T^2 - (3.8 + 0.01T)S$$

and T is temperature in degrees Celsius and S is salinity in parts per thousand (ppt).

Non-Dimensionalization of Equations

The dimensionless forms of the governing equations are used to facilitate relative magnitude comparisons of the various terms in the governing equations. The following dimensionless (star) parameters:

$$(u^*, v^*, w^*) = (u, v, wX_r/Z_r)/U_r$$

$$(x^*, y^*, z^*) = (x, y, zX_r/Z_r)/X_r$$

$$(\tau^*_x, \tau^*_y) = (\tau^w_x, \tau^w_y)/\rho_o f Z_r U_r$$

$$t^* = tf$$

$$\zeta^* = g\zeta/fU_rX_r = \zeta/S_r$$

$$\rho^* = (\rho - \rho_o)/(\rho_r - \rho_o) \qquad (9)$$

$$T^* = (T - T_o)/(T_r - T_o)$$

$$A_h^* = A_h/A_{hr}$$

$$A_v^* = A_v/A_{vr}$$

$$K_h^* = K_h/K_{hr}$$

$$K_v^* = K_v/K_{vr}$$

where (τ^w_x, τ^w_y) = wind stress in (x,y) direction, ρ_o and T_o are the expected minimum density and temperature, and ζ = water surface elevation. The dimensionless parameters resulting from these definitions are the Rossby Number R_o, Froude Number F_r, Densimetric Froude Number F_{rd}, and the horizontal and vertical Ekman Numbers E_h, E_v, respectively. These dimensionless parameters are defined as follows:

$$R_o = U_r/fX_r$$

$$Fr = U_r/(gZ_r)^{1/2}$$

$$Fr_d = \rho_o^{1/2}U_r/[gZ_r(\rho_r-\rho_o)]^{1/2} \qquad (10)$$

$$E_h = A_{hr}/fX_r^2$$

$$E_v = A_{vr}/fZ_r^2$$

All parameters with r subscripts are arbitrary reference scaling quantities.

Boundary-Fitted Equations

The CH3D-WES model utilizes a boundary-fitted or generalized curvilinear planform grid which can be made to conform to flow boundaries, providing a detailed resolution of the complex horizontal geometry of the flow system. This necessitates the transformation of the governing equations into boundary-fitted coordinates (ξ, η). If only the (x,y) coordinates are transformed, a system of equations similar to those solved by Johnson (1980) for vertically averaged flow fields is obtained. However, in the CH3D-WES model not only are the (x,y) coordinates transformed into the (ξ, η) curvilinear system but the velocity also is transformed such that its components are contravariant (i.e., perpendicular to the (ξ, η) coordinate lines). This is accomplished by employing the definitions below for the components of the Cartesian velocity (u,v) in terms of contravariant components $\bar{u}$ and $\bar{v}$

$$u = x_\xi \bar{u} + x_\eta \bar{v}$$

$$v = y_\xi \bar{u} + y_\eta \bar{v} \tag{11}$$

along with the following expressions for replacing Cartesian derivatives

$$f_x = \frac{1}{J} [(fy_\eta)_\xi - (fy_\xi)_\eta]$$

$$f_y = \frac{1}{J} [-(fx_\eta)_\xi + (fx_\xi)_\eta], \tag{12}$$

where f is an arbitrary variable and J is the Jacobian of the coordinate transformation defined as

$$J = x_\xi y_\eta - x_\eta y_\xi . \tag{13}$$

Additional metric coefficients of the transformation are:

$$G_{11} = x_\xi^2 + y_\xi^2$$

$$G_{22} = x_\eta^2 + y_\eta^2 \tag{14}$$

$$G_{12} = x_\xi x_\eta + y_\xi y_\eta = G_{21}.$$

In addition to the horizontal grid transformation, the vertical dimension is transformed into a sigma-stretched grid (Figure 1) by :

$$\sigma = \frac{z - \zeta}{\zeta + h} \tag{15}$$

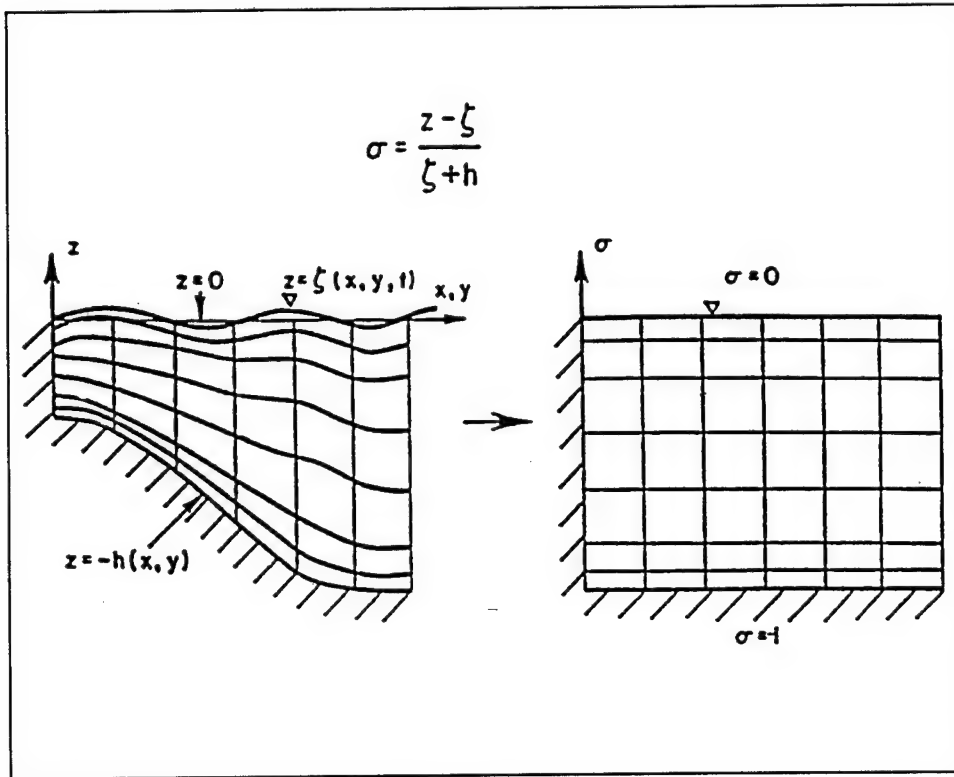where h is the water depth from the datum where $z = 0$.

Figure 1. Definition of a sigma stretched grid

With both the Cartesian coordinates and velocity components transformed, the following non-dimensional three dimensional governing equations are solved on the sigma stretched grid.

$$\frac{\partial H}{\partial t} + \frac{R_o}{J} \left[\frac{\partial}{\partial \xi} (JH\overline{u}) + \frac{\partial}{\partial \eta} (JH\overline{v}) \right] + H \frac{\partial \omega}{\partial \sigma} = o \qquad (16)$$

where

H is the total water depth, i.e., $(h + \zeta)$.

$$\frac{\partial H\overline{u}}{\partial t} = -H(\frac{G_{22}}{J^2} \frac{\partial \zeta}{\partial \xi} - \frac{G_{12}}{J^2} \frac{\partial \zeta}{\partial \eta}) + \frac{H}{J}(G_{12}\overline{u} + G_{22}\overline{v})$$

$$+ \frac{R_o x_\eta}{J^2} [\frac{\partial}{\partial \xi} (Jy_\xi H\overline{u}\,\overline{u} + Jy_\eta H\overline{u}\,\overline{v}) + \frac{\partial}{\partial \eta} (Jy_\xi H\overline{u}\,\overline{v} + Jy_\eta H\overline{v}\,\overline{v})]$$

$$- \frac{R_o y_\eta}{J^2} [\frac{\partial}{\partial \xi} (Jx_\xi H\overline{u}\,\overline{u} + Jx_\eta H\overline{u}\,\overline{v}) + \frac{\partial}{\partial \eta} (Jx_\xi H\overline{u}\,\overline{v} + Jx_\eta H\overline{v}\,\overline{v})]$$

$$- R_o \frac{\partial H\overline{u}\omega}{\partial \sigma} + \frac{E_v}{H} \frac{\partial}{\partial \sigma} (A_v \frac{\partial \overline{u}}{\partial \sigma}) + X - \text{Horizontal Diffusion}$$

$$- \frac{R_o H}{Fr_d^2} [H \int_\sigma^o (\frac{G_{22}}{J^2} \frac{\partial \rho}{\partial \xi} - \frac{G_{12}}{J^2} \frac{\partial \rho}{\partial \eta}) d\sigma$$

$$+ (\frac{G_{22}}{J^2} \frac{\partial H}{\partial \xi} - \frac{G_{12}}{J^2} \frac{\partial H}{\partial \eta})(\int_\sigma^o \rho d\sigma + \sigma \rho)] \qquad (17)$$

$$\frac{\partial H\overline{v}}{\partial t} = -H(-\frac{G_{21}}{J^2} \frac{\partial \zeta}{\partial \xi} + \frac{G_{11}}{J^2} \frac{\partial \zeta}{\partial \eta}) - \frac{H}{J}(G_{11}\overline{u} + G_{21}\overline{v})$$

$$- \frac{R_o x_\xi}{J^2} [\frac{\partial}{\partial \xi} (Jy_\xi H\overline{u}\,\overline{u} + Jy_\eta H\overline{u}\,\overline{v}) + \frac{\partial}{\partial \eta} (Jy_\xi H\overline{u}\,\overline{v} + Jy_\eta H\overline{v}\,\overline{v})]$$

$$+ \frac{R_o y_\xi}{J^2} [\frac{\partial}{\partial \xi} (Jx_\xi H\overline{u}\,\overline{u} + Jx_\eta H\overline{u}\,\overline{v}) + \frac{\partial}{\partial \eta} (Jx_\xi H\overline{u}\,\overline{v} + Jx_\eta H\overline{v}\,\overline{v})]$$

$$-R_o \frac{\partial H\bar{v}\omega}{\partial\sigma} + \frac{E_v}{H} \frac{\partial}{\partial\sigma}(A_v \frac{\partial\bar{v}}{\partial\sigma}) + Y - Horizontal\ Diffusion$$

$$+\frac{R_o H}{Fr_d^2}[H\int_\sigma^o(+\frac{G_{21}}{J^2}\frac{\partial\rho}{\partial\xi} - \frac{G_{11}}{J^2}\frac{\partial\rho}{\partial\eta})d\sigma$$

$$+(+\frac{G_{21}}{J^2}\frac{\partial H}{\partial\xi} - \frac{G_{11}}{J^2}\frac{\partial H}{\partial\eta})(\int_\sigma^o\rho d\sigma + \sigma\rho)] \tag{18}$$

From Equation 16, the vertical velocity in the sigma coordinates, ω, is computed, i.e.,

$$\omega_{top} = \omega_{bot} - \frac{\Delta\sigma}{H}\left\{\frac{\partial H}{\partial t} + \frac{R_o}{J}\left[\frac{\partial}{\partial\xi}(JH\bar{u}) + \frac{\partial}{\partial\eta}(JH\bar{v})\right]\right\} \tag{19}$$

where ω_{top} is the vertical velocity at the top of a cell and ω_{bot} is the velocity at the bottom of the cell. The computations proceed from the bottom of the water column, where $\omega_{bot} = o$, to the surface.

The vertical velocity of a water particle, w, at some σ location can then be computed from

$$w = H\omega + \frac{1+\sigma}{\beta}\frac{\partial\zeta}{\partial t} \tag{20}$$

where

$$\beta = gZ_r/(fX_r^2)$$

The horizontal diffusion terms in Equations 17 and 18 are rather lengthy and as a result are presented separately in Appendix A. The equations shown in Appendix A are a compact form of those derived by Johnson et al. 1991b. A description of the implementation of the horizontal diffusion terms is provided

in Chapman (1993). The transport equations for salt and temperature are written;

$$\frac{\partial HS}{\partial t} = \frac{E_v}{Pr_v H} \frac{\partial}{\partial \sigma} (K_v \frac{\partial S}{\partial \sigma}) - R_o \frac{\partial H\omega S}{\partial \sigma} - \frac{R_o}{J} (\frac{\partial JH\overline{u}S}{\partial \xi} + \frac{\partial JH\overline{v}S}{\partial \eta})$$

$$+ \frac{E_h}{Pr_h J} \left[\frac{\partial}{\partial \xi} \left\{ K_h H \frac{G_{22}}{J} \frac{\partial S}{\partial \xi} \right\} + \frac{\partial}{\partial \xi} \left\{ K_h H \frac{G_{12}}{J} \frac{\partial S}{\partial \eta} \right\} \right.$$

$$\left. + \frac{\partial}{\partial \eta} \left\{ K_h H \frac{G_{12}}{J} \frac{\partial S}{\partial \xi} \right\} + \frac{\partial}{\partial \eta} \left\{ K_h H \frac{G_{11}}{J} \frac{\partial S}{\partial \eta} \right\} \right]$$

and

$$\frac{\partial HT}{\partial t} = \frac{E_v}{Pr_v H} \frac{\partial}{\partial \sigma} (K_v \frac{\partial T}{\partial \sigma}) - R_o \frac{\partial H\omega T}{\partial \sigma} - \frac{R_o}{J} (\frac{\partial JH\overline{u}T}{\partial \xi} + \frac{\partial JH\overline{v}T}{\partial \eta})$$

$$+ \frac{E_h}{Pr_h J} \left[\frac{\partial}{\partial \xi} \left\{ K_h H \frac{G_{22}}{J} \frac{\partial T}{\partial \xi} \right\} + \frac{\partial}{\partial \xi} \left\{ K_h H \frac{G_{12}}{J} \frac{\partial T}{\partial \eta} \right\} \right.$$

$$\left. + \frac{\partial}{\partial \eta} \left\{ K_h H \frac{G_{12}}{J} \frac{\partial T}{\partial \xi} \right\} + \frac{\partial}{\partial \eta} \left\{ K_h H \frac{G_{11}}{J} \frac{\partial T}{\partial \eta} \right\} \right]$$

Pr_v and Pr_h are vertical and horizontal Prandtl numbers which are ratios of eddy viscosity, A_v or A_h, to eddy diffusivity, K_v and K_h.

Similarly, the transformed external mode equations are written:

$$\frac{\partial \zeta}{\partial t} + \frac{\beta}{J} [\frac{\partial}{\partial \xi} (J\overline{U}) + \frac{\partial}{\partial \eta} (J\overline{V})] \tag{23}$$

$$\frac{\partial \overline{U}}{\partial t} = -H(\frac{G_{22}}{J^2} \frac{\partial \zeta}{\partial \xi} - \frac{G_{12}}{J^2} \frac{\partial \zeta}{\partial \eta}) + \frac{1}{J} (G_{12}\overline{U} + G_{22}\overline{V})$$

$$+ \frac{R_o x_\eta}{J^2 H} [\frac{\partial}{\partial \xi} (Jy_\xi \bar{U}\bar{U} + Jy_\eta \bar{U}\bar{V}) + \frac{\partial}{\partial \eta} (Jy_\xi \bar{U}\bar{V} + Jy_\eta \bar{V}\bar{V})]$$

$$- \frac{R_o y_\eta}{J^2 H} [\frac{\partial}{\partial \xi} (Jx_\xi \bar{U}\bar{U} + Jx_\eta \bar{U}\bar{V}) + \frac{\partial}{\partial \eta} (Jx_\xi \bar{U}\bar{V} + Jx_\eta \bar{V}\bar{V})]$$

$$+ \tau_{s\xi} - \tau_{b\xi} + X - \textit{Horizontal Diffusion}$$

$$- \frac{R_o H^2}{2Fr_d^2} (\frac{G_{22}}{J^2} \frac{\partial \rho}{\partial \xi} - \frac{G_{12}}{J^2} \frac{\partial \rho}{\partial \eta}) \tag{24}$$

and

$$\frac{\partial \bar{V}}{\partial t} = -H(- \frac{G_{21}}{J^2} \frac{\partial \zeta}{\partial \xi} + \frac{G_{11}}{J^2} \frac{\partial \zeta}{\partial \eta}) - \frac{1}{J} (G_{11} \bar{U} + G_{21} \bar{V})$$

$$- \frac{R_o x_\xi}{J^2 H} [\frac{\partial}{\partial \xi} (Jy_\xi \bar{U}\bar{U} + Jy_\eta \bar{U}\bar{V}) + \frac{\partial}{\partial \eta} (Jy_\xi \bar{U}\bar{V} + Jy_\eta \bar{V}\bar{V})]$$

$$+ \frac{R_o y_\xi}{J^2 H} [\frac{\partial}{\partial \xi} (Jx_\xi \bar{U}\bar{U} + Jx_\eta \bar{U}\bar{V}) + \frac{\partial}{\partial \eta} (Jx_\xi \bar{U}\bar{V} + Jx_\eta \bar{V}\bar{V})]$$

$$+ \tau_{s\eta} - \tau_{b\eta} + Y - \textit{Horizontal Diffusion}$$

$$- \frac{R_o H^2}{2Fr_d^2} (- \frac{G_{21}}{J^2} \frac{\partial \rho}{\partial \xi} + \frac{G_{11}}{J^2} \frac{\partial \rho}{\partial \eta}) \tag{25}$$

Boundary Conditions

The boundary conditions at the free surface are

$$A_v \left[\frac{\partial \bar{u}}{\partial z}, \frac{\partial \bar{v}}{\partial z} \right] = \left(\tau_{s_\xi}, \tau_{s_\eta} \right)/\rho = \left(C\, W_\xi^2, C\, W_\eta^2 \right)$$

$$\frac{\partial T}{\partial z} = \frac{Pr_v}{E_v} K (T - T_e) \tag{26}$$

$$\frac{\partial S}{\partial z} = 0$$

whereas the boundary conditions at the bottom are

$$A_v \left[\frac{\partial \overline{u}}{\partial z} \ , \ \frac{\partial \overline{v}}{\partial z} \right] = \left(\tau_{b_\xi} \ , \ \tau_{b_\eta} \right) / \rho$$

$$= \frac{U_r}{A_{vr}} \ Z_r \ C_d \left(\overline{u}_1^2 + \overline{v}_1^2 \right)^{1/2} \left(\overline{u}_1, \ \overline{v}_1 \right) \tag{27}$$

$$\frac{\partial T}{\partial z} \ , \ \frac{\partial S}{\partial z} = 0$$

where

τ_s = wind shear stress

C = surface drag coefficient

W = wind speed

K = surface heat exchange coefficient

T_e = equilibrium temperature

τ_b = bottom shear stress

C_d = bottom friction coefficient

$\overline{u}_1$, $\overline{v}_1$ = near bottom horizontal velocity components.

Defining z_1 as one half of the bottom layer thickness and assuming a log velocity profile, C_d is given by

$$C_d = k^2 \left[\ln(z_1/z_0) \right]^{-2} \tag{28}$$

where k is the von Karman constant (0.4) and z_0 is a bottom roughness height.

Manning's formulation is employed for the bottom friction in the external mode equations if the model is used only to compute vertically averaged flow fields. The surface drag coefficient is computed according to Garratt (1977) as follows:

$$C = (0.75 + 0.067 \ W) \ x \ 10^{-3} \tag{29}$$

with the maximum allowable value being 0.003. The surface heat exchange coefficient, K, and the equilibrium temperature, T_e, are computed from meteorological data (wind speed, cloud cover, wet and dry bulb air temperatures, and relative humidity) as discussed by Edinger, Brady, and Geyer (1974). The wind speed, W, must be in meters/second.

Freshwater inflow and water temperature are prescribed along the shoreline where river inflow occurs, however, the salinity at the river boundary is specified according to a zero spatial gradient assumption (computed from the previous time step). At an ocean boundary, the water-surface elevation is prescribed along with time-varying vertical distributions of salinity and temperature. Specified values of salinity and temperature are employed during flood flow, whereas, during ebb, interior values are advected out of the grid. The normal component of the velocity and the eddy viscosity and diffusivity are set to zero along solid boundaries.

Initial Conditions

When initiating a run of CH3D-WES, the values of ζ, $\bar{u}$, $\bar{v}$, w, $\bar{U}$ and $\bar{V}$ are set to zero. Values of salinity and temperature are read from input files. These initial data are generated from prototype measurements at a limited number of locations. Once the values in individual cells are determined by interpolating from the field data, the resulting 3D field is smoothed. Generally, the salinity and temperature fields are held constant for the first few days of a simulation.

Computational Grid

A staggered grid is used in both the horizontal and vertical directions of the computational domain (Figure 2). In the horizontal direction, a unit cell consists of a ζ-point in the center ($\zeta_{i,j}$), a U-point to its left ($U_{i,j}$), and a V-point to its bottom ($V_{i,j}$). In the vertical direction, the vertical velocities are computed at the "full" grid points. Horizontal velocities, temperature, salinity, density and turbulence quantities are computed at the "half" grid points (half grid spacing below the full points).

Two arrays, each of dimension (IMAX, JMAX), are used to index the grid cells. The array NS indicates the condition of the left and right cell
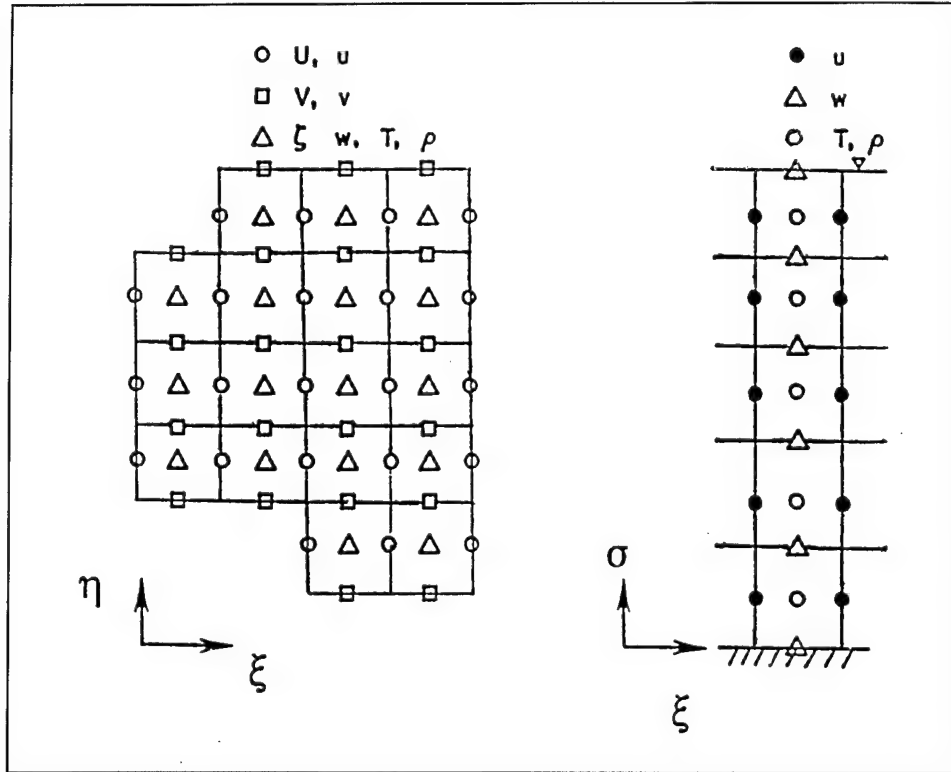
Figure 2. Staggered grid

boundaries, while the array MS denotes the condition of the top and bottom cell boundaries (Figure 3).

Numerical Solution Algorithm

Finite differences are used to replace derivatives in the governing equations, resulting in a system of linear algebraic equations to be solved in both the external and internal modes. The external mode solution consists of the surface displacement and vertically integrated contravariant unit flows $\bar{U}$ and $\bar{V}$. All of the terms in the transformed vertically-averaged continuity equation are treated implicitly, whereas, only the water-surface slope terms in the transformed vertically-averaged momentum equations are treated implicitly. If the external mode is used as purely a vertically-averaged model, the bottom friction is also treated implicitly. Those terms treated implicitly are weighted between the new and old time-steps. The resulting finite difference equations are then factored such that a ξ-sweep followed by an η-sweep of the horizontal grid yields the solution at the new time-step. Writing Equations 23-25 as
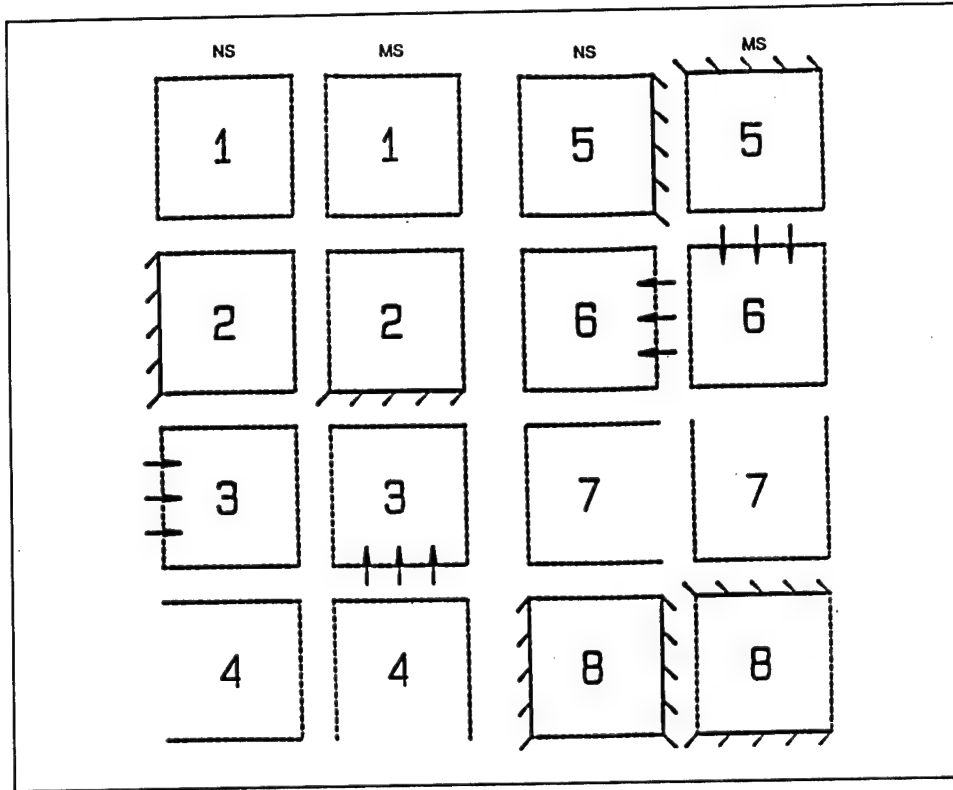
Figure 3. Computational indicators for cell boundaries

$$\frac{\partial \zeta}{\partial t} + \frac{\beta}{J} \left[\frac{\partial J\bar{U}}{\partial \xi} + \frac{\partial J\bar{V}}{\partial \eta} \right] = 0 \qquad (30)$$

$$\frac{\partial \bar{U}}{\partial t} + \frac{H}{J^2} G_{22} \frac{\partial \zeta}{\partial \xi} = M \qquad (31)$$

$$\frac{\partial \bar{V}}{\partial t} + \frac{H}{J^2} G_{11} \frac{\partial \zeta}{\partial \eta} = N \qquad (32)$$

the ξ-sweep is

$$\xi\text{-}sweep \Rightarrow \zeta^*_{ij} + \frac{\beta\theta\Delta t}{\Delta\xi J} \left[(J\bar{U})^{n+1}_{i+1,j} - (J\bar{U})^{n+1}_{ij} \right]$$

$$= \zeta_{ij}^n - (1-\theta)\frac{\beta\Delta t}{\Delta\xi J}\left[(J\bar{U})_{i+1,j}^n - (J\bar{U})_{ij}^n\right]$$

$$- \frac{\beta\Delta t}{\Delta\eta J}\left[(J\bar{V})_{i,j+1}^n - (J\bar{V})_{ij}^n\right] \tag{33}$$

and

$$\bar{U}_{ij}^{n+1} + \frac{\theta\Delta tHG_{22}}{\Delta\xi\ J^2}\left(\zeta_{ij}^* - \zeta_{i-1,j}^*\right)$$

$$= \bar{U}_{ij}^n - (1-\theta)\frac{\Delta tHG_{22}}{\Delta\xi\ J^2}\left(\zeta_{ij}^n - \zeta_{i-1,j}^n\right) + \Delta t\ M^n \tag{34}$$

The η-sweep then provides the updated ζ and $\bar{V}$ at the $n+1$ time level.

$$\eta-sweep \Rightarrow \zeta_{ij}^{n+1} + \frac{\beta\theta\Delta t}{\Delta\eta J}\left[(J\bar{V})_{i,j+1}^{n+1} - (J\bar{V})_{ij}^{n+1}\right]$$

$$= \zeta_{ij}^* - (1-\theta)\frac{\beta\Delta t}{\Delta\eta J}\left[(J\bar{V})_{i,j+1}^n - (J\bar{V})_{ij}^n\right]$$

$$+ \frac{\beta\Delta t}{\Delta\eta J}\left[(J\bar{V})_{i,j+1}^n - (J\bar{V})_{ij}^n\right] \tag{35}$$

and

$$\bar{V}_{ij}^{n+1} + \frac{\theta\Delta tHG_{11}}{\Delta\eta\ J^2}\left(\zeta_{i,j+1}^{n+1} - \zeta_{ij}^{n+1}\right)$$

$$= V_{ij}^n - (1-\theta)\frac{\Delta tHG_{11}}{\Delta\eta\ J^2}\left(\zeta_{i,j+1}^n - \zeta_{ij}^n\right) + \Delta t\ N^n \tag{36}$$

A typical value of θ of 0.55 is employed. M and N represent all terms in the equations evaluated at the previous time step.

The internal mode consists of computations for the three velocity components $\bar{u}$, $\bar{v}$, and w, salinity, and temperature. Defining the horizontal components of the 3D velocity as $\bar{u} = \dfrac{U}{H} + \bar{u}'$ and $\bar{v} = \dfrac{V}{H} + \bar{v}'$, the differential equations for the (u', v') components are obtained by subtracting the vertically averaged momentum equations (23-25) from the 3D momentum equations (17-18). This removes the water surface slope terms from the equations of motion which removes the restrictive free-surface gravity wave speed from the internal mode stability criteria.

It is important to ensure that the vertical integration of the $(\bar{u}', \bar{v}')$ is zero. This is accomplished by evaluating the nonlinear inertia and turbulent diffusion terms in the vertically-averaged momentum equations by summing the corresponding terms in the 3D equations at all vertical layers. Once $(\bar{u}', \bar{v}')$ are determined, they are slightly adjusted to absolutely ensure that their vertical sum is zero and then are added to the vertically averaged velocities to yield the horizontal components of the full 3D velocity.

The only terms treated implicitly are the vertical diffusion terms in all equations and the bottom friction in the momentum equations. Roache's (1972) second upwind differencing is used to represent the convective terms in the momentum equations, whereas, a spatially and temporally third-order scheme developed by Leonard (1979) called QUICKEST is used to represent the advective terms in Equations 21 and 22 for salinity and temperature, respectively. For example, if the velocity on the right face of a computational cell is positive then the QUICKEST value of the salinity computed for the flux through the face is

$$S_R = \frac{1}{2}\left(S_{i,j,k} + S_{i+1,j,k}\right)$$

$$-\frac{1}{6}\left[1 - \left[R_o\frac{\bar{U}_{i+1,j,k}\,\Delta t}{\Delta\xi}\right]^2\right]\left(S_{i+1,j,k} - 2\,S_{i,j,k} + S_{i-1,j,k}\right)$$

$$-\frac{1}{2}\,R_o\frac{\bar{U}_{i+1,j,k}\,\Delta t}{\Delta\xi}\left(S_{i+1,j,k} - S_{i,j,k}\right) \tag{37}$$

The more interested reader is referred to the paper by Leonard (1979).

Two Equation k - ε Turbulence Closure

A vertical k-ε turbulent eddy viscosity model which includes the effects of

wind shear, bottom shear, velocity gradient turbulence production, dissipation, diffusion and stratification has been implemented. The basic idea behind the (k-ϵ) turbulence model (Rodi, 1980; ASCE, 1988) is that the vertical eddy viscosity coefficient can be related to the turbulence energy per unit mass, k; its rate of dissipation, ϵ; and an empirical coefficient ($c_\nu = 0.09$), i.e.:

$$A_z = c_\nu \frac{k^2}{\epsilon} \tag{38}$$

The transport equations for the turbulence quantities are written:

$$\frac{\partial(Hk)}{\partial t} - \frac{1}{H}\frac{\partial}{\partial \sigma}\left[A_z \frac{\partial k}{\partial \sigma}\right] = (P_z - \epsilon + G)H \tag{39}$$

and

$$\frac{\partial(H\epsilon)}{\partial t} - \frac{1}{H}\frac{\partial}{\partial \sigma}\left[\frac{A_z}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial \sigma}\right] = (c_1\frac{\epsilon}{k}P_z - c_2\frac{\epsilon^2}{k})H \tag{40}$$

in which $\sigma_\epsilon = 1.3$, $c_1 = 1.44$, and $c_2 = 1.92$. The source and sink terms on the right hand side of equations 39 and 40 represent mechanical production of turbulence, P_z, due to vertical velocity gradients and buoyancy production or dissipation, G. The functional forms of these mechanisms are as follows:

$$P_z = \frac{A_z}{H^2}\left[G_{11}\left[\frac{\partial \overline{u}}{\partial \sigma}\right]^2 + 2G_{12}\left[\frac{\partial \overline{u}}{\partial \sigma}\frac{\partial \overline{v}}{\partial \sigma}\right] + G_{22}\left[\frac{\partial \overline{v}}{\partial \sigma}\right]^2\right] \tag{41}$$

and

$$G = \frac{A_z}{HPr_\nu}\frac{g}{\rho}\frac{\partial \rho}{\partial \sigma} \tag{42}$$

where, as previously noted, Pr_ν is the turbulent Prandtl Number. Surface and bottom boundary conditions for the turbulence quantities are specified as follows:

$$k_{s,b} = \frac{U_*^2}{\sqrt{c_v}} \tag{43}$$

and

$$\varepsilon_{s,b} = \frac{U_*^3}{\dfrac{\kappa H \Delta \sigma}{2}} \tag{44}$$

where κ is the von Karman coefficient. The friction velocity, U_*, at the surface boundary is defined as the square root of the resultant wind shear stress, TSR, where:

$$TSR = G_{11}\tau_x^2 + 2G_{12}\tau_x\tau_y + G_{22}\tau_y^2 \tag{45}$$

in which τ_x and τ_y are the components of the wind stress. The bottom friction velocity is computed in an identical way with the wind vectors replaced by the contravariant velocity components. The suppression of the vertical diffusivity by stratification is accomplished by modifying the computed value as follows:

$$K_z = A_z(1+3R_i)^{-2} \tag{46}$$

where R_i is the Richardson's Number (Bloss et al. 1988).

$$R_i = \frac{\dfrac{g}{\rho H}\dfrac{\partial \rho}{\partial \sigma}}{\dfrac{1}{H^2}\left[\dfrac{\partial\sqrt{\overline{u}^2 + \overline{v}^2}}{\partial \sigma}\right]^2} \tag{47}$$

A complete description of the implementation of the k-ϵ model is presented in Chapman (1994).

3 Structure of Sigma CH3D-WES

The CH3D-WES model has a main program as well as several subroutines. Subroutines governing model setup are called from the main program while subroutines governing computations are called from subroutine CH3DM2. Each of these is listed below with a description of its function. Entry points in subroutines are also noted. Two INCLUDE files, application.inc and ch3d.inc, are needed. They are used to set up parameters, dimensions of various arrays, and COMMON blocks. During model compilation, these files are inserted wherever the INCLUDE statements in the source code call for the files. Several input data files are required. These are listed in Appendix C.

CH3D	The main program.
CH3DIR	Reads data from main input file, FILE 4 (see Appendix B), which controls computations, input, and output. Various constants are computed, and the vertical (σ-) layer thicknesses are set.
CH3DTR	Reads (x,y) coordinates (ft) and depths (ft) at the cell corners of the boundary-fitted grid from FILE 15 (ITRAN=2). The coordinates are then multiplied by the scale factor, XMAP, and divided by XREF to make them nondimensional. Subroutine BJINTR is called to provide the coordinate derivatives needed to compute the metrics of the transformation.
BJINTR	Computes various coordinate derivatives and sets the water depths HU(I,J) and HV(I,J) on the faces of each computational cell.
CH3DIH	Prints water depths, if requested by input data. Also, the water depths are made nondimensional by dividing by ZREF.
CH3DND	Normalizes several variables and parameters, such as the Ekman number, Rossby number, time-step, etc.

CH3DII   Sets up the arrays of boundary flags that indicate the nature of computational cell boundaries. In addition, arrays controlling the computation of the convective terms in the momentum equations and the water surface cross-derivative terms are set up. One-dimensional channel cells are identified.

CH3DIF   Initializes various variables for a cold start run and opens time series output files for elevation, velocities, salinity, etc. as well as print and snapshot files. The hot start capability is not operational.

CH3DIV   The arrays created in CH3DII concerning water surface cross-derivatives contain logical values. Those arrays are used in this subroutine to create arrays containing numerical values. These arrays, i.e. AFV1(I,J), etc. are used to control computation of not only water surface cross-derivatives but other variables as well.

CH3DWS   Controls the reading of either wind speed or wind stress. If the wind speed is read, the stress is computed from Garratt's equation. ENTRY CH3DWT controls the time-varying reads and computations.

        The subroutines above are called from CH3D in the sequence given. Before calling CH3DM2, which controls the computations, the initial salinity field is read from FILE 74. The initial temperature field is read from FILE 17 and made dimensionless.

CH3DM2   Final subroutine called from CH3D. All subroutines controlling the actual 3D computations are called from this subroutine in the order they appear below.

CH3DDP   Computes the total water depths from the latest water surface elevation field. ENTRY CH3DDM sets total water depths at the intermediate time level M and ENTRY CH3DDN sets total water depths at time level N.

CH3DTK   Reads equilibrium temperatures and surface heat exchange coefficients from FILE 19 and then casts them into nondimensional form. ENTRY CH3DTB controls the time-varying read and interpolation.

CH3DRI   Reads river inflows from FILE 13. ENTRY CH3DRV controls the time-varying read and interpolation.

CH3DTI   Reads and initializes tidal boundary conditions from FILE 16. ENTRY CH3DTD updates boundary values.

CH3DSAI	Reads salinity and temperatures at tidal boundaries from FILE 76. ENTRY CH3DSAV controls the time-varying reads, interpolation, and conversion to nondimensional form.
CH3DTEI	Reads temperatures at river inflow boundaries from FILE 78. ENTRY CH3DTEV controls the reading of time-varying temperatures and interpolation.
CH3DDE	Computes the water densities using Equation (8). The baroclinic terms in the momentum equations are then evaluated.
CH3DKE	Computes the eddy viscosity and eddy diffusivity coefficients.
CH3DWT	ENTRY in CH3DWS for reading time-varying wind data.
CH3DTB	ENTRY in CH3DTK for reading time-varying equilibrium temperature and heat-exchange coefficient.
CH3DRV	ENTRY in CH3DRI for reading time-varying river flows.
CH3DTEV	ENTRY in CH3DTEI for reading time-varying temperatures at river inflow boundaries.
CH3DTD	ENTRY in CH3DTI for reading time-varying tide data.
CH3DSAV	ENTRY in CH3DSAI for reading time-varying salinities and temperatures at tidal boundaries.
CH3DDN	ENTRY in CH3DDP for assigning total water depths at time level N.
CH2DXY	Computes the vertically averaged flow field from the vertically averaged equations of motion.
CH3DDP	Using the water surface field computed in CH2DXY, computes the total water depths at time level $N+1$.
CH3DXYZ	Computes the 3D velocity field. Mass conservation is ensured by forcing the vertical sum of the horizontal components of the 3D velocity to match the vertically integrated values computed in CH2DXY.
CH3DDI	Computes the convective and diffusion terms in the momentum equations using the most recent computation results from CH3DDP and CH3DXYZ. These terms are then employed at the next time step in CH2DXY and CH3DXYZ.
CH3DSA	Computes the salinity field.

CH3DTE   Computes the temperature field.

CH3DBL   Checks the water surface elevations for the program "blowing up".

CH3DOT   Controls the output printed and/or written to files for plotting. Output is in terms of physical dimensional variables. Subroutine CH3DC1 is called with ENTRIES CH3DC2, CH3DC3, CH3DC4, CH3DC5, CH3DC6, CH3DC7, CH3DC8, CH3DC9, CH3DCA, CH3DCC, CH3DCD, and CH3DCE. Each is described below.

CH3DC1   Provides dimensional water surface elevations.

CH3DC2   Provides dimensional physical vertically-averaged velocity in x-direction.

CH3DC3   Provides dimensional physical vertically-averaged velocity in y-direction.

CH3DC4   Provides dimensional physical horizontal velocity component in x-direction.

CH3DC5   Provides dimensional physical horizontal velocity component in y-direction.

CH3DC6   Provides dimensional physical vertical component of 3D velocity.

CH3DC7   Provides salinity.

CH3DC8   Provides dimensional temperature.

CH3DC9   Provides dimensional physical magnitude and direction of horizontal velocity.

CH3DCA   Provides dimensional physical horizontal components of 3D velocity at the *centers* of cells.

CH3DCC   Provides dimensional water density.

CH3DCD   Provides dimensional vertical eddy viscosity.

CH3DCE   Provides dimensional vertical eddy diffusivity.

In subroutine CH3DOT, the following files are created for use in generating time series plots, vector plots, or contour plots.

FILE 21	For time series plots of dimensional water surface elevation at specified horizontal locations.
FILE 22	For time series plots of dimensional, Cartesian horizontal velocities (x and y directions) at cell centers at specified horizontal locations.
FILE 23	Geometry of study area (needed for plotting snapshots or contours).
FILE 24	For velocity vector plots and contour plots of surface elevation, salinity, temperature, etc.
FILE 25	For time series plots of discharges at specified horizontal ranges.
FILE 31	For time series plots of salinity at specified horizontal locations in all layers.
FILE 34	For time series plots of temperature at specified horizontal locations in all layers.
FILE 35	For time series plots of vertical eddy viscosity at specified horizontal locations in all layers.
FILE 36	For time series plots of vertical eddy diffusivity at specified horizontal locations in all layers.
FILE 37	For time series plots of density at specified horizontal locations in all layers.

As previously indicated, there are two INCLUDE files, application.inc and ch3d.inc, needed for running the model. Of these, application.inc is used to set up the model parameters for running the particular application. The following parameters are set. They are used to dimension arrays in COMMON blocks in ch3d.inc and other arrays in the model. Of these, ICELLS, JCELLS, IJMAX, and KM have to be set exactly. The others can be greater than or equal to what is needed. The ch3d.inc file does not have to be changed from application to application, but remains the same.

ICELLS	: Number of grid cells in the ξ-direction
JCELLS	: Number of grid cells in the η-direction
IJMAX	: The greater of ICELLS and JCELLS, plus 1
KM	: Number of σ-layers in the vertical
NSTATS	: Maximum number of gage stations where information will be saved
NTIDES = 11	: Not used
NRIVRS	: Number of river boundaries used

NCNST = 37 : Maximum number of tidal constituents used (set to 37) - used only if tidal signals were generated using constituents - not operational.

NBNDS   : Number of open water boundaries used

NBARRS   : Number of interior thin-wall barriers

NPRWIN   : Number of print windows for printing model results

NSNAPS   : Number of snapshot windows where information is saved

NRANGS   : Number of ranges where discharge information is saved

NTIDFN   : Number of tide functions used

NTIDBN   : Number of tidal boundaries used

NTIDPT   : Maximum number of values in the input tide functions

NROWS   : Maximum number of computational chains used in ξ-direction

NCOLS   : Maximum number of computational chains used in η-direction

KROWS   : Larger of NROWS and NCOLS, plus 1

NX8PTS   : Number of one-cell wide channel cells in ξ-direction

NY8PTS   : Number of one-cell wide channel cells in η-direction

SPVAL   : A small value to which the vertical eddy coefficients, etc. are set as a default

4 Summary

The purpose of this report is to describe the main features of the sigma version of the CH3D-WES hydrodynamic model. In Chapter 2, the basic governing equations are given followed by the boundary and initial conditions employed. The report outlines the structure of the computer model in Chapter 3, listing the names of the various subroutines, their functions, and the calling sequence. This should help users who are interested in following the logic of the model. Also listed are various output files created by the model and their contents.

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Appendix A
Transformed Horizontal
Momentum Diffusion Terms

X - Horizontal Diffusion

$$= \frac{Y_\eta}{J^2} \left[\frac{A_h G_{22}}{J} \left[(X_\xi H\overline{u})_\xi + (X_\eta H\overline{v})_\xi \right] \right]_\xi$$

$$+ \frac{Y_\eta}{J^2} \left[\frac{A_h G_{11}}{J} \left[(X_\xi H\overline{u})_\eta + (X_\eta H\overline{v})_\eta \right] \right]_\eta$$

$$- \frac{X_\eta}{J^2} \left[\frac{A_h G_{11}}{J} \left[(Y_\xi H\overline{u})_\eta + (Y_\eta H\overline{v})_\eta \right] \right]_\eta$$

$$- \frac{Y_\eta}{J^2} \left[\frac{A_h G_{12}}{J} \left[(X_\xi H\overline{u})_\eta + (X_\eta H\overline{v})_\eta \right] \right]_\xi$$

$$- \frac{Y_\eta}{J^2} \left[\frac{A_h G_{12}}{J} \left[(X_\xi H\overline{u})_\xi + (X_\eta H\overline{v})_\xi \right] \right]_\eta$$

$$+ \frac{X_\eta}{J^2} \left[\frac{A_h G_{12}}{J} \left[(Y_\xi H\overline{u})_\eta + (Y_\eta H\overline{v})_\eta \right] \right]_\xi$$

$$+ \frac{X_\eta}{J^2} \left[\frac{A_h G_{12}}{J} \left[(Y_\xi H\overline{u})_\xi + (Y_\eta H\overline{v})_\xi \right] \right]_\eta$$

Y - Horizontal Diffusion

$$= \frac{X_\xi}{J^2} \left[\frac{A_h G_{11}}{J} \left[(Y_\eta H\overline{v})_\eta + (Y_\xi H\overline{u})_\eta \right] \right]_\eta$$

$$- \frac{Y_\xi}{J^2} \left[\frac{A_h G_{11}}{J} \left[(X_\eta H\overline{v})_\eta + (X_\xi H\overline{u})_\eta \right] \right]_\eta$$

$$+ \frac{X_\xi}{J^2} \left[\frac{A_h G_{22}}{J} \left[(Y_\eta H\overline{v})_\xi + (Y_\xi H\overline{u})_\xi \right] \right]_\xi$$

$$- \frac{Y_\xi}{J^2} \left[\frac{A_h G_{22}}{J} \left[(X_\eta H\overline{v})_\xi + (X_\xi H\overline{u})_\xi \right] \right]_\xi$$

$$- \frac{X_\xi}{J^2} \left[\frac{A_h G_{12}}{J} \left[(Y_\eta H\overline{v})_\eta + (Y_\xi H\overline{u})_\eta \right] \right]_\xi$$

$$- \frac{X_\xi}{J^2} \left[\frac{A_h G_{12}}{J} \left[(Y_\eta H\overline{v})_\xi + (Y_\xi H\overline{u})_\xi \right] \right]_\eta$$

$$+ \frac{Y_\xi}{J^2} \left[\frac{A_h G_{12}}{J} \left[(X_\eta H\overline{v})_\eta + (X_\xi H\overline{u})_\eta \right] \right]_\xi$$

$$+ \frac{Y_\xi}{J^2} \left[\frac{A_h G_{12}}{J} \left[(X_\eta H\overline{v})_\xi + (X_\xi H\overline{u})_\xi \right] \right]_\eta$$

Replacing $H\overline{u}$ and $H\overline{v}$ with $\overline{U}$ and $\overline{V}$, respectively, the same expressions apply in the external mode equations.

Appendix B
List of Input Data in File 4

DUMMY
TITLE   Run description (Format A80)

DUMMY
IT1, IT2, DT, ISTART, ITEST, ITSALT (2I8,F8.0,4I8)
   IT1 ; Starting time step (always set = 1)
   IT2 ; Ending time step
   DT ; Computational time step in sec
   ISTART = 0   ; Cold start
      > 0   ; Hot start (not operational)
   ITEST = 0   ; No diagnostic output
      > 0   ; Diagnostic output
   ITSALT   ; Number of time steps after which salinity and
      temperature computations are initiated

DUMMY
WPRCRD (9I8,A8)
   WPRCRD   ; Number of print control cards which follow
DUMMY
WXCEL1, WXCEL2, WYCEL1, WYCEL2, WZCEL1, WZCEL2, WPRINT,
  WPRSTR, WPREND, WPRVAR (9I8,A8)

If WPRCRD > 0, WPRCRD cards have to be furnished below.

    WXCEL1   ; Starting ξ-cell index
    WXCEL2   ; Ending ξ-cell index
    WYCEL1   ; Starting η-cell index
    WYCEL2   ; Ending η-cell index
    WZCEL1   ; Starting sigma layer index
    WZCEL2   ; Ending sigma layer index
    WPRINT   ; Printing interval (number of time steps)
    WPRSTR   ; Time step when printing starts
    WPREND   ; Time step when printing ends
    WPRVAR   ; Character string indicating variables printed

Note : The following characters are used in WPRVAR for designating different variables.

E : Surface elevation (cm)
X : X-direction unit flow rate (cm^2/sec)
Y : Y-direction unit flow rate (cm^2/sec)
U : X-direction velocity (cm/sec)
V : Y-direction velocity (cm/sec)
W : Z-direction velocity (cm/sec)
S : Salinity (ppt)
T : Temperature (deg C)
A : Average velocity magnitude (cm/sec) and direction
    (measured clockwise from the true North, deg)

DUMMY
SNPCRD (9I8,A8)
   SNPCRD   ; Number of snapshot control cards to follow
DUMMY
SXCEL1, SXCEL2, SYCEL1, SYCEL2, SZCEL1, SZCEL2, SNPINT,
  SNPSTR, SNPEND, SNPVAR (9I8,A8)
If SNPCRD > 0, SNPCRD cards have to be furnished below.

   SXCEL1   ; Starting ξ-cell index
   SXCEL2   ; Ending ξ-cell index
   SYCEL1   ; Starting η-cell index
   SYCEL2   ; Ending η-cell index
   SZCEL1   ; Starting sigma layer index
   SZCEL2   ; Ending sigma layer index
   SNPINT   ; Snapshot interval (number of time steps)
   SNPSTR   ; Time step when snapshots start
   SNPEND   ; Time step when snapshots end
   SNPVAR   ; Character string indicating snapshot
                 variables (same notation is used as in
                 WPRVAR)

DUMMY
NRANG (9I8,A8)
   NRANG   ; Number of ranges for computing discharges
DUMMY
RANGDR, RPOS1, RPOS2, RPOS3, RRNAME (7X,A1,3I8,A45)
If NRANG > 0, NRANG cards have to be furnished below.
   RANGDR ; Range direction (X for ξ and Y for η)
   RPOS1   ; ξ (η) cell index of range line
   RPOS2   ; Starting η (ξ) cell index for range
   RPOS3   ; Ending η (ξ) cell index for range
   RRNAME ; Range description (name)

DUMMY

IGI, IGH, IGT, IGS, IGU, IGW, IGC, IGQ, IGP (10I8) : Printout flags. A
value of 1 turns printing on and 0 turns it off.

    IGI ; Print arrays such as NS, MS, NR, MR, etc.

    IGH   ; Print all depth arrays

    IGT = 0   ; Not used

    IGS = 0   ; Print restart arrays

    IGU = 0   ; Not used

    IGW = 0   ; Not used

    IGC   ; Print grid coordinates and depths

    IGQ = 0   ; Not used

    IGP ; Save grid information in FILE 23 for plotting
                 snapshots

DUMMY
XREF, ZREF, UREF, COR, GR, RO0, ROR, T0, TR (10F8.0)

    XREF   ; Reference horizontal grid distance
                (Maximum horizontal dimension divided by
                number of cells in that direction, cm)

    ZREF   ; Reference depth (average depth in cm)

    UREF   ; Reference horizontal velocity
                (average velocity in cm/sec)

    COR   ; Coriolis parameter

    GR   ; Gravitational acceleration (cm/sec^2)

    RO0   ; Minimum density expected (gm/cc)

    ROR   ; Reference density (maximum expected) (gm/cc)

    T0   ; Minimum temperature (Celsius)

    TR   ; Reference temperature (maximum expected)
                (Celsius)

DUMMY
THETA (10F8.0)

    THETA   ; Time level weighting factor in computations
                $(0.5 \leq \text{THETA} \leq 1.0)$

DUMMY
ITEMP, ISALT, ICC, IFI, IFA, IFB, IFC, IFD (10I8)

    ITEMP = 0   ; No computation of temperature

          = -1 ; Compute temperature (use daily equilibrium
                temperature as river boundary temperature)

          = -2 ; Compute temperature (use time-varying
                temperature as river boundary temperature)

    ISALT = 0   ; No computation of salinity

          = -2 ; Compute salinity, setting salinity and
                temperature at tidal boundaries

    ICC = 0   ; Not used

    IFI = 1 ; Compute nonlinear (inertia) terms

       = 0   ; No computation of nonlinear terms

    IFA = 0 ; Not used

    IFB = 0 ; Not used

```
IFC   =  0  ; Not used
IFD   =  1  ; Compute horizontal diffusion terms
      =  0  ; No computation of horizontal diffusion terms
```

DUMMY
TWE, TWH, FKB (3F8.0)

TWE	; Temperature in the epilimnion (for computing initial conditions)
TWH	; Temperature in the hypolimnion (for computing initial conditions)
FKB	; Vertical grid index of the initial thermocline location (for computing initial conditions)

Note: The initial conditions computed using TWE, TWH, and FKB are overridden by FILE 17.

DUMMY
IEXP, IAV, AVR, AV1, AV2, AVM, AVM1, AHR (2I8,8F8.0)

IEXP	: Vertical eddy coefficient flag
IEXP = 0	; Constant eddy coefficient.
= 1	; k - ϵ turbulence closure
IAV = 0	; Not used
AVR	; Reference vertical eddy viscosity (cm^2/sec)
AV1	; Not used
AV2	; Not used
AVM1	; Minimum allowable vertical eddy diffusivity (cm^2/sec)
AHR	; Reference horizontal eddy viscosity or diffusivity (cm^2/sec)

DUMMY
GAMAX, GBMAX (2F8.0)

GAMAX	; Maximum value of eddy viscosity (cm^2/sec)
GBMAX	; Maximum value of eddy diffusivity (cm^2/sec)

DUMMY
IWIND, TAUX, TAUY (I8,5F8.0)

IWIND = 0	; Steady and uniform wind stress
= 1	; Steady and uniform wind speed
= 2	; Time variable and uniform wind stress
= 3	; Time variable and uniform wind speed
TAUX	; Uniform wind stress in x-direction if IWIND=0
	Uniform wind speed in x-direction if IWIND=1
TAUY	; Uniform wind stress in y-direction if IWIND=0
	Uniform wind speed in y-direction if IWIND=1

DUMMY

ISPAC(I), I=1,10 (10I8)
   ISPAC(1)   = 0 ; Constant Mannings n = RSPAC(1)
             = 1 ; Read Mannings n from File 18
  ISPAC(2 - 3) = 0 ; Not used
   ISPAC(4)   = 1 ; Flag for computing open boundary velocities
   ISPAC(5 - 10) = 0 ; Not used

DUMMY
JSPAC(I), I=1,10 (10I8)

   JSPAC(1)   = 0 ; Not used
   JSPAC(2)   ; Flag for 3-D mode, quadratic friction
            = 0 ; Constant bottom friction factor = CTB
            = 1 ; Bottom friction based on logarithmic law
   JSPAC(3)   ; Flag for Coriolis terms
            = 0 ; Coriolis effects accounted for
            = -1 ; Coriolis effects neglected
   JSPAC(4 - 10) = 0 ; Not used

DUMMY
RSPAC(I), I=1,10 (10F8.0)
  RSPAC(1)   ; Constant Mannings n
   RSPAC(2 - 10) = 0. ; Not used

DUMMY
IBTM, HADD, HMIN, H1, H2, SSS0, HMAX (I8,5F8.0)
   IBTM   ; Bottom bathymetry flag
   IBTM = 0 ; Bottom depth varies linearly from west to
               east of the basin
      = 1   ; Bottom depth varies linearly from south to
               north of the basin
      = 2   ; Bottom depth array for cell center depths
               read from input file (FILE 4)
       = 3   ; Bottom depth arrays HS, HU, HV read from
               FILE 12
       = 4   ; Bottom depths and coordinates of cell corners
               read from FILE 15 (set ITRAN=2)
   HADD   ; A constant depth added to the depth array
               (cm)
   HMIN   ; Minimum water depth (cm)
   H1   ; Bottom depth (cm) along the west or south
               boundary of the basin for IBTM = 0 or 1
   H2   ; Bottom depth (cm) along the east or north
               boundary of the basin for IBTM = 0 or 1
   SSS0   ; Initial water surface elevation (cm)
   HMAX   ; Maximum water depth (cm) allowed

DUMMY
ISMALL, ISF, ITB, ZREFBN, CTB, BZ1, ZREFTN, TZ1 (3I8,7F8.0)

```
         ISMALL = 0       ; Small amplitude assumption is invoked.
                            Surface elevation is not added to the still
                            water depth to obtain the total depth
                = 1  ; Small amplitude assumption is not invoked.
                       Surface elevation is added to the still
                       water depth to obtain the total depth
  ISF    = 0     : Not used
  ITB : Bottom friction flag
            = 1  ; Linear bottom friction for internal mode
            > 1  ; Quadratic bottom friction for internal mode
  ZREFBN     ; Reference height above bottom (cm)
  CTB        ; Constant bottom drag coefficient (typical
               value 0.003)
  BZ1        ; Bottom roughness height (cm)
  ZREFTN     ; Reference height at the top (cm)
  TZ1        ; Constant surface roughness height (cm)
```

DUMMY
XMAP, ALXREF, ALYREF (10F8.0)

```
  XMAP        ; Mapping factor that scales the (x,y)
                coordinates created by the grid generation
                code to the real world
  ALXREF    ; X-reference length in the computational plane
  ALYREF    ; Y-reference length in the computational plane
```

Note : ALXREF and ALYREF are used if ITRAN= 0

DUMMY
ITRAN (10I8)

```
  ITRAN  = 0        ; Cartesian grid
         = 1  ; Curvilinear grid created by WESCOR.  Cell
                 corner coordinates rad from FILE 15
         = 2  ; Curvilinear grid created by WESCORA.  Cell
                 corner coordinates and depths read from
                 FILE15
```

DUMMY
ITBRK(I), I=1,10 (10I8)

```
  ITBRK(I), I=1,10;  ; Time steps at which information is written to
                hot-start files (increasing order)
```

DUMMY
NSTA, NFREQ, NSTART (10I8)

```
  NSTA        ; Number of stations where information is saved
                for time series plots of currents
  NFREQ     ; Time step interval for saving currents
  NSTART    ; Beginning time step for saving currents
```

DUMMY

IST(K), JST(K), STATID(K) (2I4,A48)
If NSTA > 0, NSTA cards have to be furnished below.
   IST(K), JST(K)   ; Cell indices (I,J) of a station where
                      currents are saved
   STATID(K)   ; Station description

DUMMY
NSTAS, NFREQS, NSTRTS (10I8)
   NSTAS   ; Number of stations where water surface
             elevations are saved for time series plots
   NFREQS   ; Time step interval for saving water surface
             elevations
   NSTRTS   ; Beginning time step for saving water surface
             elevations

DUMMY
ISTS(K), JSTS(K), STATS(K) (2I4,A48)
If NSTAS > 0, NSTAS cards have to be furnished below.
   ISTS(K),JSTS(K)   ; Cell indices (I,J) of a station where water
                  surface elevations are saved
   STATS(K) ; Station description

DUMMY
MSTA, MFREQ, MSTART (10I8)
   MSTA   ; Number of stations where salinity and
             temperature information is saved for time
             series plots
   MFREQ   ; Time step interval for saving information
   MSTART   ; Beginning time step for saving information

DUMMY
ISTSA(K), JSTSA(K), STATSA(K) (2I4,A48)
If MSTA > 0, MSTA cards have to be furnished below.
   ISTSA(K),JSTSA(K) ; Cell indices (I,J) of a station where
                  salinity and temperature are saved
   STATSA(K)   ; Station description

DUMMY
NRIVER   ; Number of river boundaries (2I8,F8.0,4I8)
   NRIVER = 0   ; No river boundaries
        < 0   ; River inflows are steady
        > 0   ; Time variable inflows

If NRIVER = 0, use the following cards

   DUMMY
   DUMMY

If NRIVER > 0, use the following cards

DUMMY
IJRDIR(K), IJRROW(K), IJRSTR(K), IJREND(K)* (10I8)

IJRDIR(K) = 1   ; River boundary is on left (west)
           = 2   ; River boundary is on bottom (south)
           = 3   ; River boundary is on right (east)
           = 4   ; River boundary is on top (north)
IJRROW(K)   ; Index of the row (J) or column (I)
                  of the river boundary
IJRSTR(K) ; Starting I or J index of the river boundary
IJREND(K) ; Ending I or J index of the river boundary
* NRIVER cards have to be furnished

If NRIVER < 0, use the following cards

DUMMY
IJRDIR(K), IJRROW(K), IJRSTR(K), IJREND(K)* (10I8)

IJRDIR(K) = 1   ; River boundary is on left (west)
           = 2   ; River boundary is on bottom (south)
           = 3   ; River boundary is on right (east)
           = 4   ; River boundary is on top (north)
IJRROW(K)   ; Index of the row (J) or column (I)
                  of the river boundary
IJRSTR(K) ; Starting I or J index of the river boundary
IJREND(K) ; Ending I or J index of the river boundary
*|NRIVER| cards have to be furnished followed by the cards
shown below.

DUMMY
ICELL, JCELL, QRIVER(K,IJ)* (2I8,F8.0,4I8)
    ICELL, JCELL   ; Coordinates of a cell (I,J) where QRIVER is
                  prescribed
    QRIVER(K,IJ)   ; Steady river inflow
*Repeat for all the river cells, in order.

DUMMY
NBAR (10I8)
    NBAR   ; Number of interior thin-wall barriers

If NBAR = 0, use the following card
    DUMMY

If NBAR > 0, use the following cards

DUMMY
IJBDIR(K), IJBROW(K), IJBSTR(K), IJBEND(K)* (10I8)

IJBDIR(K) = 1   ; Barrier is in ξ-direction
           = 2   ; Barrier is in η-direction
IJBROW(K)   ; Index of row (J) or column (I) of barrier
IJBSTR(K) ; Starting I or J index of barrier
IJBEND(K) ; Ending I or J index of barrier
*NBAR cards have to be furnished.

DUMMY
TIDFNO, TIDBND (10I8)
    TIDFNO   ; Number of tidal elevation tables entered as
input
    TIDBND   ; Number of tidal elevation boundaries

DUMMY
If TIDFNO > 0, read the following card(s)
TIDSTR(I), I=1,TIDFNO (10I8)
    TIDSTR(I) ; The entry number in each tidal elevation
                 table corresponding to the starting time of
                 the simulation

DUMMMY
If TIDBND > 0, TIDBND cards of the following format have to be read.
IJDIR(I), IJROW(I), IJSTR(I), IJEND(I), TIDTYP(I), TIDFN1(I),
TIDFN2(I) (4I8,A8,5I8)
    IJDIR(I) = 1   ; Tidal boundary is on left (west)
              = 2   ; Tidal boundary is on bottom (south)
              = 3   ; Tidal boundary is on right (east)
              = 4   ; Tidal boundary is on top (north)
    IJROW(I)   ; Index of the row (J) or column (I) of the
                    tidal boundary
    IJSTR(I)   ; Starting I or J index of the tidal boundary
    IJEND(I)   ; Ending I or J index of the tidal boundary
    TIDTYP(I) = "CONSTANT" ; Constant tidal elevation between
                    IJSTR(I) and IJEND(I)
              = "INTERP " ; Linear interpolation of tidal
                    elevation between IJSTR(I) and
                    IJEND(I)
    TIDFN1(I)   ; The number of the tidal elevation table for
                    CONSTANT or INTERP type of boundaries
    TIDFN2(I)   ; The number of the second tidal elevation
                    table used for interpolation on INTERP type
                    boundaries

Optional input:

DUMMY
I,J (free format)   ; Indices of a cell where HS is reset to 0.

DUMMY
I,J (free format)   ; Indices of a cell where HU is reset to 0.

DUMMY
I,J (free format)   ; Indices of a cell where HV is reset to 0.

DUMMY
I,J, RDEPTH (free format)   ; Indices and depth (ft) of a cell where
                    HS is reset to non-zero value RDEPTH.

Appendix C
List of Input Data Files

FILE 13

River inflows are read from FILE 13. These data are read first as a time line (DAY and HOUR) formatted by 2I8. Next, the (I,J) location and discharge in cubic feet per second for each cell of each river boundary are read and formatted by (2I8, F8.0).

FILE 14

Wind data are read from FILE 14. These data are in the form of time (DAY and HOUR) and the x and y components of the wind velocity in meters per second of each wind field used. These data are formatted by (2I5,6F10.0).

FILE 15

The (x,y) coordinates and depths of the grid cell corners are read from FILE 15. This file is created from a run of the grid generation code WESCORA and a depth interpolation program. The first line contains the file name formatted as A80. The number of corner points in ξ and η are read next unformatted. The coordinates and depths are read next unformatted, one line per corner.

FILE 16

Tabular tide data are read from FILE 16. The first line is the title formatted as A80. The tide data are in the form of time (MONTH, DAY, YEAR, HOUR, MINUTES) and the water surface elevations in centimeters relative to selected datum for TIDFNO points. These data are formatted by (I2,1X,2I3,1X,2I2, (T17,8F8.2)).

FILE 17

The initial temperature field in degrees Celsius is read from FILE 17 by format (10E12.5). This file is created from a few observed values. The resulting field is then smoothed in the ξ and η directions several times before it is written to FILE 17.

FILE 18

A field of Manning's n values may be input by format (20F4.0). The input values are multiplied by 0.001 in the source code to yield the actual values. They are input by rows.

FILE 19

Daily average equilibrium temperatures in degrees Celsius and surface heat exchange coefficients in units of cm/sec are read from FILE 19. These data are in the form of time (DAY and HOUR), equilibrium temperature, and heat exchange coefficient. They are formatted by (2I5,F10.0,E12.5).

FILE 74

The initial salinity field in parts per thousand is read from FILE 74 by format (10E12.5). This file is created in the same fashion as FILE 17.

FILE 76

Time-varying salinity in parts per thousand and temperature in degrees Celsius at tidal boundaries are read from FILE 76 if salinity and temperature are to be computed. These data are in the form of time (DAY and HOUR) formatted by (2I5). Next, the (I,J) location of each tidal boundary cell and the vertical distribution of salinity, starting from the top layer to the bottom layer are read. These data are followed by temperature data using the same format as for the salinity. The format is (2I5,11F5.0)

FILE 78

Time-varying temperature data at river flow boundaries are read from FILE 78 if temperatures are to be computed and equilibrium temperatures are not used as river boundary temperatures. These data are in the form of a time (DAY and HOUR) formatted by (2I5). Next, the (I,J) location of river flow boundary cells and corresponding temperatures starting from top layer to bottom layer are read. These data are formatted by (2I5,11F6.0).

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13. ABSTRACT *(Maximum 200 words)*

A time-varying three-dimensional (3-D) numerical hydrodynamic, salinity, and temperature model called CH3D-WES has been developed. The water surface, 3-D velocity field, and 3-D salinity and temperature fields are computed. Major physical processes affecting circulation and vertical mixing of a large water body are modeled. A particular feature of the model is the solution of transformed equations on a boundary-fitted grid in both the horizontal and vertical planes. The horizontal grid is a general nonorthogonal curvilinear grid, whereas the vertical grid is normally referred to as a sigma-stretched grid.

This user's guide presents a detailed discussion of the theoretical aspects of the 3-D model (e.g., basic equations, boundary conditions, turbulence closure, etc.). This discussion is followed by a discussion of the organization of the computer code and input data requirements.

14. SUBJECT TERMS

Boundary-fitted	Numerical model	Temperature
Curvilinear	Salinity	Three-dimensional
Hydrodynamics	Sigma-stretched	Turbulence

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